Foundation of Computer Science — FM2

Assignment 6 on Turing machines and (un)decidability

Watch the video lectures 16, 17, 18 and 19 of Week 5.

In what follows, Turing machines are given as tuples $A = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, where Q is the set of its states, Σ is the input alphabet, Γ is the tape alphabet ($\Sigma \subset \Gamma$), δ is the transition function, $q_0 \in Q$ is the start state, $B \in \Gamma \setminus \Sigma$ is the blank symbol, and $F \subseteq Q$ is the sets of final states. If δ is represented by a transition table, then the head row displays the states while the head column contains the tape symbols.

1. Given the Turing machine

$$M = (\{z_0, z_0', z_1, z_1', z_2, q\}, \{a\}, \{a, b, *\}, \delta, z_0, *, \{q\}),$$

where δ is defined by

	z_0	z_1	z_0'	z_1'	z_2
$*$	$(z_0, *, R)$	$(q, *, R)$	$(z_2, *, L)$	$(z_1', *, R)$	$(z_0, *, R)$
a	(z_1, a, R)	(z_0', b, R)	(z_1', a, R)	(z_0', b, R)	(z_2, a, L)
b	(z_0, b, R)	(z_1, b, R)	(z_0', b, R)	(z_1', b, R)	(z_2, b, L)

Determine the language accepted by accepting state and by halting.

2. Give a deterministic Turing machine accepting the language

$$\{ a^n b^n c^n \mid n \geq 0 \}.$$

The choice of the acceptance mode is left to you.

3. Design a nondeterministic Turing machine *deciding* the language

$$\{ w_1\#w_2\#\ldots\#w_n \mid n \geq 2,\ w_i \in \{a, b\}^+, 1 \leq i \leq n,\ \text{there is } k \geq 2 \text{ such that } w_k = w_1 \}.$$