Foundation of Computer Science — FM2

Assignment 4 on the video lectures about context-free grammars

Watch the video lectures 9, 10, 11 of Week 3.

In what follows, a context-free grammar will be given in the form $G = (N, T, P, S)$, where N is the set of non-terminals (variables), T is the set of terminals, P is the set of productions, and $S \in N$ is the start symbol.

1. Prove that the following languages are context-free but not regular:

   (a) $\{ ww^R \mid w \in \{a, b\}^* \}$, where w^R denotes the mirror image of w that is inductively defined by

   $$\begin{array}{rcl} \varepsilon^R & = & \varepsilon \\ (va)^R & = & av^R \text{ with } v \in \Sigma^*,\ a \in \Sigma \end{array}$$

   (b) $\{ w1^n \mid w \in \{0, 1\}^*, |w| = n \}$,

2. Give a context-free grammar for each of the following languages:

   (a) $\{ a^m b^n \mid 0 \leq m \leq n \}$

   (b) $\{ a^i b^j c^k \mid i \neq j \text{ or } j \neq k \}$

   Which of your grammars are unambigious? Can you tell which of the languages is inherently ambigious?

3. Given the context-free grammar

   $$G = (\{S\}, \{a, b\}, \{S \to SS, S \to aaSb, S \to bSaa, S \to \varepsilon\}, S).$$

   Does

   $$L(G) = \{w : w \in T^*, |w|_a = 2 \cdot |w|_b\}$$

   hold?

4. Construct a context-free grammar in Chomsky normal form that is equivalent to

   $G = (\{S, A, B, C\},\ \{b\},\ \{S \to ABC,\ S \to AB,\ A \to b,\ B \to Bb,\ B \to \varepsilon,\ C \to BB\},\ S).$